A Double-Well System Composed of Phonons in a Pair of Trapped Ions

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14. ABSTRACT

This report presents a theoretical framework using a double-well system to describe the transverse phonon coupling between a pair of trapped ions. The double-well system is a special case of the more generalized Bose-Hubbard Model, with exactly two sites available to store particles. Analogous double-well systems in solid-state junctions and in Bose-Einstein condensates have been used for interferometric sensing, as well as quantum information processing, and many exotic quantum phenomena such as coherent tunneling and the Josephson Effect also arise naturally in such systems. This perspective offers insight into possible methods to control phonons in trapped ions, which are commonly used as a resource for quantum computation and quantum simulation.

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1. Introduction

Double-well potentials have extensive applications in many branches of physics and have resulted in devices that are useful for precision sensing, as well as quantum information processing. An example of a physical system containing a double-well potential is a Josephson junction containing a thin insulating barrier between two superconductors (*1*), and superconducting quantum interference devices (SQUIDs) based on superconducting loops containing Josephson junctions have been used as sensitive magnetometers (*2*). SQUIDs can also be used to store qubits for quantum computation (*3*). In atomic physics, recent demonstrations of matter-wave interference with Bose-Einstein condensates in double-well traps have also shown great promise for applications in precise inertial and gravitational field sensing (*4*).

Given the usefulness of double-well systems, this report will examine the transverse phonon coupling between a pair of trapped ions and show a direct mapping to a double-well system. Such a comparison has never been presented in the literature previously, but should be extremely valuable in gaining an understanding of what quantum behaviors can be expected and how to control and manipulate such a system. This system is of interest to many quantum applications since transverse phonons in trapped ions are now regularly used in the laboratory as a data bus for quantum computation and for quantum simulation (*5*). Previous work has already shown that the transverse phonons in a long string of trapped ions can be modeled by the Bose-Hubbard Hamiltonian (*6*). This report will follow the same methodology to analyze the special case with exactly two ions in the trap.

2. Transverse Phonons in a Pair of Trapped Ions

Let us consider a pair of trapped ions confined by an oscillating RF field in a Paul trap exerting mutually repulsive Coulomb force on each other, as shown in figure 1. The time-averaged potential around the equilibrium points is harmonic in the lowest order, with a corresponding Hamiltonian

$$H = \sum_{i=1}^{2} \frac{p_i^2}{2m} + \frac{1}{2} m \sum_{i=1}^{2} \left(\omega_x^2 x_i^2 + \omega_y^2 y_i^2 + \omega_z^2 z_i^2 \right) + \frac{e^2}{\sqrt{(z_1 - z_2)^2 + (x_1 - x_2)^2 + (y_1 - y_2)^2}}$$

where the first term is the kinetic energy of the ions, the second term is the trapping potential, and the last term is the Coulomb interaction. p_i and x_i are the momentum and the displacement of the i-th ion, respectively, and ω_x, ω_y, ω_z are the harmonic frequencies of the trap in the respective x, y, and z directions. The ions line up along the weakest trapping (axial) direction z,

and the trapping potential is stronger in the orthogonal (transverse) directions x and y. For simplicity, we require that the trapping frequencies in all three directions are non-degenerate, so that motion along x, y, or z do not couple to one another.
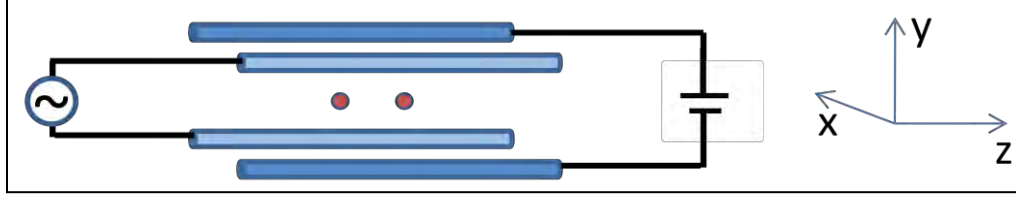

Figure 1. Two ions in a linear RF Paul trap. Phonons are the ion's vibrational excitation in a harmonic potential, and are coupled between the ions by the Coulomb force.

Here we will focus only on vibration in one particular transverse direction x. At low temperature, the ion displacement from equilibrium point is small compared to the distance between ions, and under realistic experimental conditions, the Coulomb energy is small compared to the potential energy. Phonons are defined in the usual way, as in the i-th ion has n_x phonons in the x direction if the vibrational state of the i-th ion is in the n_x-th Fock state $|n_{x,i}\rangle$ in the corresponding harmonic potential. The transverse phonon potential in the x direction can be approximated to lowest order by

$$H_x = \hbar \omega'_x \left(a_1^\dagger a_1 + a_2^\dagger a_2 \right) + \hbar J \left(a_1^\dagger a_2 + a_2^\dagger a_1 \right),$$

where

$$\omega'_x = \left(1 - \frac{1}{2} \frac{e^2/(m\omega_x^2)}{|z_1^0 - z_2^0|^3} \right) \omega_x$$

$$J = \frac{1}{2} \frac{e^2/(m\omega_x^2)}{|z_1^0 - z_2^0|^3} \omega_x$$

The term ω'_x accounts for the spatially dependent shift of the trapping frequency, $\hbar J$ is the tunneling energy, and z_i^0 is the equilibrium position in z of the i-th ion. The operators $a_i^\dagger$ and a_i are the creation and annihilation operators defined by the relation $x_i = \sqrt{\hbar / m\omega_x} \, (a_i + a_i^\dagger)$. The first term of the Hamiltonian H_x is the energy associated with the number of phonons in each site, according the number operator $n_{x,i} = a_i^\dagger a_i$. The second term of the Hamiltonian is the tunneling energy, where a phonon is destroyed from one site and created in the other site by the operators $a_i^\dagger a_j$. In the basis $\binom{n_{x,1}}{n_{x,2}}$, the Hamiltonian H_x is represented by the matrix

$$H_x = \hbar \begin{pmatrix} \omega'_x & J \\ J & \omega'_x \end{pmatrix}$$

The phonons in the two sites (double-well) are coupled by the off-diagonal term $\hbar J$, and an initial phonon population in one site will oscillate between the two sites at frequency $2J$ (7).

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This is directly analogous to Rabi oscillation in a two-level system or to Josephson oscillation where there is a barrier between two superconductors.

3. Experimental Considerations

The transverse phonon energy $\hbar\omega_x$ for ions in an RF Paul trap depends on the amplitude of the oscillating electric fields at the ions' locations, and can be tuned by changing the RF power and electrode distances. The coupling strength $\hbar J$ depends on the strength of the axial confinement, which, for a linear trap, can be tuned with DC voltages at endcap electrodes. Let us consider two different regimes: (i) a decoupled system, where the tunneling energy is negligible compared to phonon energy, i.e., $J \ll \omega_x$; and (ii) a strongly-coupled system, where the tunneling energy is significant compared to phonon energy, i.e. $J \sim \omega_x$. In the first case, the ions are far apart from each other such that the Coulomb interaction is small compared to the phonon energy. Phonons can be created or destroyed independently at each site using Raman sideband transitions (8), assuming the phonon excitations can be coherently driven on a time scale much faster than h/J. The internal spin of the ions is typically used for adding exactly a fixed number of phonons to the system at a time (9), but coherent states can also be generated using a σ_x or σ_z force (10). Individual addressing of the ions should be trivial given that the separation between the ions is large in this regime. In the second case, the strong coupling between the phonons at two sites means that tunneling cannot be ignored. By diagonalizing the Hamiltonian, we find that the stationary states are comprised of the collective motion of both ions and have energies $\omega'_x \pm J$, which are respectively labeled as the center-of-mass mode for the symmetric motion and the stretch mode for the antisymmetric motion. In this regime, it is best to directly excite these common modes of motion using Raman sideband transition, instead of exciting the uncoupled phonon modes since the tunneling time is on the order of the sideband transition time and will make clean transitions to the uncoupled phonon modes difficult.

The system can, in principle, switch between the two cases previously described by ramping the axial trap strength, and with negligible unintended transverse phonon excitation in the adiabatic regime, where during the transition the rate of change in the Hamiltonian is much less than the transverse phonon excitation energy, i.e. $d\omega'_x/dt \ll \omega'^2_x$. The major difference between the trapped ion system and other double-well systems such as Josephson junctions and Bose-Einstein condensates is that phonons in a harmonic trap do not interact with each other and, therefore, has no additional energy cost to place multiple phonons at the same site. This is in sharp contrast to electrons or neutral atoms, which do interact with each other and will result in an energy shift that depends on the number of particles located at the same site. With neutral atoms, it is possible to turn off atom-atom interaction through a Feshbach resonance, but requires additional effort to reach high magnetic fields with fine tuning and high stability. Therefore, we expect a Rabi-like oscillation in the phonon population of each site in the trapped ion system, while

previous experiments with Bose-Einstein condensates in a double-well have observed a macroscopic quantum self-trapping due to inter-particle interaction (*11*).

For a more direct analogy between trapped ions and other double-well systems, on-site energy shift can be artificially generated by adding anharmonicity to the trapping potential or by inducing a light shift at the ions' locations, as described in reference (*6*). In such a setup, where tunneling $\hbar J$ and on-site energy $\hbar U a_i^{\dagger 2} a_i^2$ can be independently controlled, if the system transforms from the strongly coupled regime to the decoupled regime *adiabatically* with respect to both J and U, we expect to observe phonon number squeezing analogous to the superfluid to Mott-Insulator quantum phase transition predicted for a long string of ions (*6*). This phenomenon has already been observed with neutral bosonic atoms in a double-well optical lattice (*12*), where $2N$ atoms initially in a single well are adiabatically transformed to an equal number of atoms N in each site of the double-well. Using the same principle, we also expect to create a coherent state of phonons in the two sites if the system is transformed from the strongly coupled regime to the decoupled regime *diabatically*. This phenomenon also arises from the Bose-Hubbard theory (*13*) and has been observed in neutral atoms (*12*).

4. Conclusion

This report has shown a direct mapping of the transverse phonon coupling between two trapped ions to a double-well system, and described how to experimentally access some of the rich quantum phenomenology in trapped ions that arises from double-well physics. Besides being able to control the phonons through number squeezing, further studies of this system may yield other potential quantum applications.

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